

Goldstein.

1.2. By definition $\vec{R} = \frac{1}{M} \sum_i m_i \vec{r}_i$

$$M^2 \vec{R}^2 = \left[\sum_i m_i \vec{r}_i \right] \left[\sum_j m_j \vec{r}_j \right]$$

$$= \sum_{i,j} m_i m_j (\vec{r}_i \cdot \vec{r}_j)$$

$$r_{ij}^2 = (\vec{r}_i - \vec{r}_j)^2 = r_i^2 + r_j^2 - 2(\vec{r}_i \cdot \vec{r}_j)$$

$$\frac{1}{2} [-r_{ij}^2 + r_i^2 + r_j^2] = \vec{r}_i \cdot \vec{r}_j$$

$$\Rightarrow M^2 R^2 = \sum_{i,j} \frac{1}{2} [r_i^2 + r_j^2 - r_{ij}^2] m_i m_j$$

$$= \frac{1}{2} \sum_{i,j} r_i^2 m_i m_j + \frac{1}{2} \sum_{i,j} r_j^2 m_i m_j - \frac{1}{2} \sum_{i,j} r_{ij}^2 m_i m_j$$

$$= \boxed{M \sum_i m_i r_i^2 - \frac{1}{2} \sum_{i,j} m_i m_j r_{ij}^2}$$